Multi-scalar field cosmology from SFT: an exactly solvable approximation

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Abstract

We consider the appearance of multiple scalar fields in SFT inspired non-local models with a single scalar field at late times. In this regime all the scalar fields are free. This system minimally coupled to gravity can be analyzed approximately or numerically. The main result of this note is the introduction of an exactly solvable model which obeys an exact solution in the cosmological context for the Friedmann equations and that reproduces the behavior expected from SFT in the asymptotic regime. Different applications of such a potential to multi-field cosmological models are discussed.

1 Introduction

In this note we briefly review a new class of cosmological models based on string field theory (SFT) (for details see reviews [1]) and p-adic string theory [2]. We introduce an exactly solvable model reproducing the behavior of the initial model in the asymptotic regime (see [3, 4, 5, 6] and references therein for a more detailed analysis on the subject). It is known that SFT and p-adic string theory are UV-complete. One can therefore expect that the resulting (effective) models should be free of pathologies. Furthermore, models originating from SFT exhibit one general non-standard property: they have terms with infinitely many derivatives, i.e. non-local terms. Higher derivative terms usually produce the well known Ostrogradski instability [7]. However, the Ostrogradski result is related to derivatives order higher than two but finite. In the case of infinitely many derivatives it is possible that instabilities do not appear.

Contemporary cosmological observational data [8, 9] strongly support that the present Universe exhibits an accelerated expansion providing thereby an evidence for a dominating DE component [10]. Recent results of WMAP [9] together with the data on Ia supernovae give the following bounds for the DE state parameter $w_{\rm DE} = -1.02^{+0.14}_{-0.16}$. Note that the present cosmological observations do not exclude an evolving DE state parameter w. Non-local models of the type obtained from SFT may have effective phantom behavior and can therefore be interesting for the present cosmology. To construct a stable model with $w < -1$ one should construct from the fundamental theory, which is stable and admits quantization, an effective theory with the Null Energy Condition (NEC) violation. This is a hint towards SFT inspired cosmological models.

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2 Model setup

We are interested in non-local models arising from SFT in large-time regime . Our starting point is the action

$$S=\int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N}+\frac{1}{g_o^2}\left(-\frac{1}{2}\partial_\mu T\partial^\mu T+\frac{1}{2\alpha'}T^2-\frac{1}{\alpha'}v(\bar{T})\right)-\Lambda'\right). \tag{1}$$

We work in $1+3$ dimensions with signature $(-,+,+,+)$, the coordinates are denoted by Greek indices $\mu,\nu,\dots$, running from 0 to 3. G_N is the Newtonian constant, $8\pi G_N=1/M_P^2$, where M_P is the Planck mass, α' is the string length squared (we do not assume $\alpha'=1/M_P^2$), g_o is the open string coupling constant and Λ' is a constant term. $\bar{T}=\mathcal{G}(\alpha'\Box)T$, with $\Box=D^\mu\partial_\mu=\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$ and D_μ is the covariant derivative, T is a scalar field primarily associated with the open string tachyon. The function $\mathcal{G}(\alpha'\Box)$ can be non-polynomial, thus producing a manifest non-locality. Fields are dimensionless while $[g_o]=$ length. $v(\bar{T})$ is the open string tachyon self-interaction. It does not have a term quadratic in T. Factor $1/\alpha'$ in front of the tachyon potential looks unusual and can be easily removed by a rescaling of fields. For our purposes it is convenient to keep all the fields dimensionless.

Such a four-dimensional action is motivated by string field theory (see [4] for details). In SFT one has $\mathcal{G}(\alpha'\Box)=e^{-\frac{\beta}{2}\alpha'\Box}$, where β is a parameter determined exclusively by the conformal field theory of the string, but we keep this function general. We stress once again that appearance of non-localities is a general feature of SFT based models and it is exactly the feature that we are going to explore here[1].

Introducing the field $T_b=\bar{T}$ and dimensionless coordinates $\bar{x}_\mu=x_\mu/\sqrt{\alpha'}$, we can rewrite the above action as

$$S=\int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N}+\frac{1}{g_o^2}\left(-\frac{1}{2}\partial_\mu\tilde{T}_b\partial^\mu\tilde{T}_b+\frac{1}{2}\tilde{T}_b^2-v(T_b)\right)-\Lambda'\right). \tag{2}$$

where we omit bars for simplicity and set hereafter $\alpha'=1$. We emphasize that the potential of the field T_b is $V=-\frac{1}{2}T_b^2+v(T_b)$. Assuming an extremum of the potential V exists, one can linearize the theory around it using $T_b=T_0-\tau$. As a result one obtains

$$V=-\frac{1}{2}\tau^2+\frac{v(T_0)''}{2}\tau^2+V(T_0).$$

The action (2), linearized around the extremum of the potential, can be written as

$$S=\int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N}+\frac{1}{2g_o^2}\tau\mathcal{F}(\Box)\tau-\Lambda\right), \tag{3}$$

where $\mathcal{F}(\Box)=(\Box+1)\mathcal{G}^{-2}(\Box)-m^2$ with $m^2\equiv v(T_0)''$ and $\Lambda=\Lambda'+\frac{V(T_0)}{g_o^2}$. In SFT one would have $\mathcal{F}_{\rm SFT}(\Box)=(\Box+1)e^{\beta\Box}-m^2$. $\mathcal{F}$ is in fact the inverse propagator and it is natural to expect $\beta<0$ in the SFT case corresponding to convergent propagator at large momenta. From this SFT example we can draw a very important lesson. Assume $m=0$. Then $\mathcal{F}_{\rm SFT}=(\Box+1)e^{\beta\Box}$ and the non-locality does not show up at all. Indeed, the poles of the propagator do not feel the exponent. Another way of thinking is that the exponential factor can be eliminated by a field redefinition. The situation is dramatically different for $m\neq 0$. The propagator $1/\mathcal{F}_{\rm SFT}$ has infinitely many poles. This is a manifestation of the non-locality. Also note that the physics would be totally different for full function $\mathcal{F}$ and its truncated series expansion because the pole structure may get modified considerably.

A very important role is played by the spectrum of the theory, determined by the equation

$$\mathcal{F}(J)=(J+1)\mathcal{G}^{-2}(J)-m^2=0. \tag{4}$$

We call it *characteristic* equation. It can be an algebraic or a transcendental. We consider $\mathcal{F}(\Box)$ of general form, with the only assumption that all roots are simple. The analyticity of the function $\mathcal{F}$ on the complex

[1]The appearance of higher derivatives is not an exclusive feature of this theory. Non-commutative theories, for instance, also have higher derivative, but these non-local structures are very different.

plane is also important for representing $\mathcal{F}$ by the convergent series expansion:

$$\mathcal{F}=\sum_{n=0}^{\infty} f_n \Box^n\,, \quad f_n\in\mathbb{R}. \tag{5}$$

Reality of coefficients is required by the hermiticity of the Lagrangian. Strictly speaking even the analyticity requirement can be weakened, but in this case one has to be careful with poles and the corresponding convergence domain of the series.

Classical solutions of the equation of motion were studied and analyzed in [4]. The key point in the analysis is the fact that action (3) is fully equivalent to the action

$$S_{local}=\int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N}-\frac{1}{g_o^2}\sum_i\frac{\mathcal{F}'(J_i)}{2}\left(g^{\mu\nu}\partial_\mu\tau_i\partial_\nu\tau_i+J_i\tau_i^2\right)-\Lambda\right) \tag{6}$$

with many free local scalar fields. Here J_i are roots of the characteristic equation (4) and there are as many scalar fields as roots of the characteristic equation. The details of the equivalence statement can be found in [4, 6, 11].

It is important that the roots J_i as well as the coefficients $\mathcal{F}'(J_i)$ can be complex. This do not represent a problem, since all the local scalar fields are just mathematical functions without physical interpretation. What is important is that the original field τ must be real since it represents a physical excitation. This is simple to achieve. The roots J_i are either real or complex conjugate. Complex conjugate J_i would yield complex conjugate (up to an overall constant factor) solutions for τ_i. Thus, making the linear combination of all τ_i real is just a matter of choosing the integration constants appropriately. Finally, note that the coefficients $\mathcal{F}'(J_i)$ can be eliminated by a field rescaling.

In [6] it was proven that cosmological perturbations in the free theory with one non-local scalar field (3) and in the corresponding local theory (6) with many scalar field are equivalent as well. Moreover, several examples of evolution and perturbations were developed. The difficulty here, however, is that not many exactly solvable models with multiple scalar fields are known. Studying perturbations without having an exact solution is not an easy task. Of course, there is no need to have infinitely many scalar fields. On the contrary, on can set almost all of them to zero by choosing trivial integration constants. The point is that the models for which exact solutions are known are mostly models with one single field. Reducing the action (6) to a single local field completely misses the rich and non-trivial structure coming from SFT. The most intriguing case of complex masses requires at least two fields. In fact, we need the original function τ to be real and thus one cannot keep only one complex field without its complex conjugate.

3 Exactly solvable model

An exact analytic solution for equations of motion following from the action (6) (and furthermore (3)) is not known. However, it is much more transparent to work with exact solutions rather than with asymptotics when one wants to study cosmological perturbations. There is a chance to modify the potential such that: first, the model becomes exactly solvable and, second, all the new terms vanish rapidly in the regime of interest, so that the previous picture is restored. Furthermore an exactly solvable model in cosmology has its own value just because not so many exactly solvable models are known. In our particular case we deal with many scalar fields and this complicates the problem. Moreover we have complex coefficients in the Lagrangian and this is an unexplored problem.

We consider the following modified action

$$\begin{aligned}S=\int d^4x\sqrt{-g}\Bigg[\frac{R}{16\pi G_N}+\frac{1}{g_o^2}\Bigg(-\sum_i\frac{\mathcal{F}'(J_i)}{2}\Big(g^{\mu\nu}(\partial_\mu\tau_{i+}\partial_\nu\tau_{i+}+\partial_\mu\tau_{i-}\partial_\nu\tau_{i-})+\\ +J_i(\tau_{i+}^2+\tau_{i-}^2)\Big)-\frac{3\pi G}{2}\left(\sum_i\mathcal{F}'(J_i)(\alpha_{i+}\tau_{i+}^2+\alpha_{i-}\tau_{i-}^2)\right)^2+c.c.\Bigg)-\Lambda\Bigg],\end{aligned} \tag{7}$$

where $H_0=\sqrt{\frac{8\pi G_N\Lambda}{3}}$, $G\equiv G_N/g_o^2$ is a dimensionless analog of the Newton constant and $\alpha_{i\pm}$ are the two solutions[2]of $J_i=-\alpha_i(\alpha_i+3H_0)$. To simplify the notation, we introduce new indeces $P,Q,R\dots$ taking the degeneracy in α_i into account. Namely, we consider $P,Q,R\dots=i_+,i_-,j_+\dots$, bearing in mind that one can now have $J_P=J_Q$ for some values of P and Q. With this piece of notation the modified action (7) takes the form

$$S=\int d^4x\sqrt{-g}\left[\frac{R}{16\pi G_N}+\frac{1}{g_o^2}\left(-\tilde{\sum_P}\frac{\mathcal{F}'(J_P)}{2}\left(g^{\mu\nu}\partial_\mu\tau_P\partial_\nu\tau_P+J_P\tau_P^2\right)-\frac{3\pi G}{2}\left(\tilde{\sum_P}\mathcal{F}'(J_P)\alpha_P\tau_P^2\right)^2\right)-\Lambda\right] \quad (8)$$

where $\tilde{\sum}_P$ indicates that complex conjugate quantities are included in the sum[3].

We now specialize to the case of spatially flat Friedmann–Robertson–Walker (FRW) Universe, with metric

$$ds^2=-dt^2+a^2(t)\left(dx_1^2+dx_2^2+dx_3^2\right), \quad (9)$$

where $a(t)$ is the scale factor and t is the cosmic time. The Hubble parameter, as usual, is $H=\dot{a}/a$ and the dot here and hereafter in this paper denotes a derivative with respect to t. Background solutions for τ are therefore assumed to be spacially homogeneous as well. The local action (8) now admits an exact solution to its equations of motion, which are

$$\begin{aligned}3H^2&=4\pi G\left(\tilde{\sum_P}\mathcal{F}'(J_P)\left(\dot{\tau}_P^2+J_P\tau_P^2\right)+3\pi G\left(\tilde{\sum_P}\mathcal{F}'(J_P)\alpha_P\tau_P^2\right)^2\right)+8\pi G_N\Lambda\,,\\ \dot{H}&=-4\pi G\tilde{\sum_P}\mathcal{F}'(J_P)\dot{\tau}_P^2\,,\end{aligned} \quad (10)$$

and

$$\ddot{\tau}_P+3H\dot{\tau}_P+J_P\tau_P+6\pi G\alpha_P\tau_P\tilde{\sum_Q}\mathcal{F}'(J_Q)\alpha_Q\tau_Q^2=0, \quad \text{for all } P\,. \quad (11)$$

One can explicitly check that there is the following solution[4]

$$\begin{aligned}\tau_P&=\tau_{P0}e^{\alpha_P t}\,,\\ H&=H_0-2\pi G\tilde{\sum_P}\mathcal{F}'(J_P)\alpha_P\tau_{P0}^2e^{2\alpha_P t}\,.\end{aligned} \quad (12)$$

This solution is valid for any number of fields (including single field model) and for any values of parameters α_P (i.e. real, complex, etc.). Moreover, we see that if $\mathrm{Re}(\alpha_P)<0$ for all P then the quartic term in the scalar fields potential vanishes and we are left with free fields. Thus for large times the model (6) is restored and we can speak about the asymptotic regime of SFT based models. Stability of the solution deserves a deeper analysis and methods used in [13] can be applied.

4 Cosmological perturbations

In this section we want to present the equations describing cosmological perturbations for the exact solution (12).

[2]In the special case $\frac{4J}{9H_0^2}=1$ the two solutions coincide and we are left with just one scalar field.

[3]For example $\tilde{\sum}_P J_P\tau_P^2=\sum_P\left(J_P\tau_P^2+J_P^*\tau_P^{2*}\right)$.

[4]Similar modification of the action was considered in [12] and the analysis in the case of real roots was performed.

For the action (8) one has the following perturbation equations for many scalar fields:

$$
\begin{aligned}
&\ddot{\zeta}_{PQ}+\dot{\zeta}_{PQ}\left(3H+\frac{\ddot{\tau}_P}{\dot{\tau}_P}+\frac{\ddot{\tau}_Q}{\dot{\tau}_Q}\right)+\zeta_{PQ}\left(-3\dot{H}+\frac{k^2}{a^2}\right)=\left[\frac{\tau_P}{\dot{\tau}_P}\left(J_P+6\pi G\alpha_P\tilde{\sum_R}\mathcal{F}'(J_R)\alpha_R\tau_R^2\right)\right.\\
&\left.-\frac{\tau_Q}{\dot{\tau}_Q}\left(J_Q+6\pi G\alpha_Q\tilde{\sum_R}\mathcal{F}'(J_R)\alpha_R\tau_R^2\right)\right]\left(\tilde{\sum_S}\mathcal{F}'(J_S)\frac{\dot{\tau}_S^2}{\rho+p}\left(\dot{\zeta}_{PS}+\dot{\zeta}_{QS}\right)+\frac{2\varepsilon}{1+w}\right)+\\
&+12\pi G\tilde{\sum_R}\mathcal{F}'(J_R)\alpha_R\tau_R\dot{\tau}_R\left(\alpha_P\frac{\tau_P}{\dot{\tau}_P}\zeta_{PR}-\alpha_Q\frac{\tau_Q}{\dot{\tau}_Q}\zeta_{QR}\right), \qquad (13)
\end{aligned}
$$

$$
\begin{aligned}
&\ddot{\varepsilon}+\dot{\varepsilon}H\left(2-6w+3c_s^2\right)+\varepsilon\left(\dot{H}(1-3w)-15wH^2+9H^2c_s^2+\frac{k^2}{a^2}\right)=\\
=&\frac{k^2}{a^2}\frac{1}{\rho}\frac{2}{\rho+p}\tilde{\sum_{R,S}}\left(J_R+6\pi G\alpha_R\tilde{\sum_P}\mathcal{F}'(J_P)\alpha_P\tau_P^2\right)\mathcal{F}'(J_R)\mathcal{F}'(J_S)\tau_R\dot{\tau}_R\dot{\tau}_S^2\zeta_{RS}\,. \qquad (14)
\end{aligned}
$$

Here $\zeta_{PQ}=\frac{\delta\tau_P}{\dot{\tau}_P}-\frac{\delta\tau_Q}{\dot{\tau}_Q}$ is the gauge invariant variable for the scalar fields perturbation and ε is the gauge invariant total energy density perturbation and k is the comoving wavenumber (see, for instance, [14, 15, 16, 6] for a derivation of perturbation equations and details). The collective energy density and pressure are

$$
\rho=\frac{1}{2}\tilde{\sum}_P\mathcal{F}'(J_P)\dot{\tau}_P^2+V\,,\qquad p=\frac{1}{2}\tilde{\sum}_P\mathcal{F}'(J_P)\dot{\tau}_P^2-V\,,
$$

with

$$
V=\frac{1}{2}\tilde{\sum}_P\mathcal{F}'(J_P)J_P\tau_P^2+\frac{3\pi G}{2}\left(\tilde{\sum}_P\mathcal{F}'(J_P)\alpha_P\tau_P^2\right)^2.
$$

We also introduced the following notation $w\equiv p/\rho$ for the equation of state parameter and $c_s^2\equiv\dot{p}/\dot{\rho}$ for the speed of sound . Using the explicit solution (12) one finds

$$
\begin{aligned}
\rho&=\frac{1}{2}\tilde{\sum_P}\mathcal{F}'(J_P)\tau_{0P}^2e^{2\alpha_Pt}\left(\alpha_P^2+J_P\right)+\frac{3\pi G}{2}\left(\tilde{\sum_P}\mathcal{F}'(J_P)\alpha_P\tau_{0P}^2e^{2\alpha_{P_i}t}\right)^2+g_0^2\Lambda=\frac{3}{8\pi G}H^2\,,\\
p&=\frac{1}{2}\tilde{\sum_P}\mathcal{F}'(J_P)\tau_{0P}^2e^{2\alpha_Pt}\left(\alpha_P^2-J_P\right)-\frac{3\pi G}{2}\left(\tilde{\sum_P}\mathcal{F}'(J_P)\alpha_P\tau_{0P}^2e^{2\alpha_Pt}\right)^2-g_0^2\Lambda=-\frac{\dot{H}}{4\pi G}-\frac{3}{8\pi G}H^2\,,\\
w&=\frac{\tilde{\sum}_P\mathcal{F}'(J_P)\tau_{0P}^2e^{2\alpha_Pt}\left(\alpha_P^2-J_P-3\pi G\alpha_P\tilde{\sum}_Q\mathcal{F}'(J_Q)\alpha_Q\tau_{0Q}^2e^{2\alpha_Qt}\right)-g_0^2\Lambda}{\tilde{\sum}_P\mathcal{F}'(J_P)\tau_{0P}^2e^{2\alpha_Pt}\left(\alpha_P^2+J_P+3\pi G\alpha_P\tilde{\sum}_Q\mathcal{F}'(J_Q)\alpha_Q\tau_{0Q}^2e^{2\alpha_Qt}\right)+g_0^2\Lambda}=-1-\frac{2}{3}\frac{\dot{H}}{H^2}\,,\\
c_s^2&=\frac{\tilde{\sum}_P\mathcal{F}'(J_P)\alpha_P\tau_{0P}^2e^{2\alpha_Pt}\left(\alpha_P^2-J_P-6\pi G\tilde{\sum}_Q\mathcal{F}'(J_Q)\alpha_Q^2\tau_{0Q}^2e^{2\alpha_Qt}\right)}{\tilde{\sum}_P\mathcal{F}'(J_P)\alpha_P\tau_{0P}^2e^{2\alpha_Pt}\left(\alpha_P^2+J_P+6\pi G\tilde{\sum}_Q\mathcal{F}'(J_Q)\alpha_Q^2\tau_{0Q}^2e^{2\alpha_Qt}\right)}=-1-\frac{\ddot{H}}{3\dot{H}H}\,.
\end{aligned}
$$

Substituting the explicit solution for τ_P in (13) one notices that the term given by the second derivative of the potential in the r.h.s. cancels with the one proportional to $\dot{H}$ in the l.h.s. Equation (14) evaluated on the exact solution gets simplified because the contribution coming from the quartic term in the potential drops in this case. One can express these two equations in terms of H and its derivatives and also eliminate J_P form all these expression by replacing it with the combination $-\alpha_P(\alpha_P+3H_0)$. All the H_0 contributions coming form this substitution simply drops because of the symmetry properties of the expressions involving

J_P.

$$\ddot{\zeta}_{PQ} + \dot{\zeta}_{PQ}\left(3H + \alpha_P + \alpha_Q\right) + \zeta_{PQ}\frac{k^2}{a^2} = \\ = \frac{1}{\dot{H}}\left(\alpha_P - \alpha_Q\right)\left(4\pi G \tilde{\sum_R} \mathcal{F}'(J_R)\tau_{0R}^2\alpha_R^2 e^{2\alpha_R t}\left(\dot{\zeta}_{PR} + \dot{\zeta}_{QR}\right) + 3H^2\varepsilon\right), \quad (15)$$

$$\ddot{\varepsilon} + \dot{\varepsilon}\left(5H + 4\frac{\dot{H}}{H} - \frac{\ddot{H}}{\dot{H}}\right) + \varepsilon\left(6H^2 + 14\dot{H} + 2\frac{\dot{H}^2}{H^2} - 3H\frac{\ddot{H}}{\dot{H}} + \frac{k^2}{a^2}\right) = \\ = \frac{k^2}{a^2}\frac{4}{3}\frac{(4\pi G)^2}{\dot{H}H^2}\tilde{\sum_{R,S}}\mathcal{F}'(J_R)\mathcal{F}'(J_S)\alpha_R\alpha_R^2\alpha_S^2\tau_{0R}^2\tau_{0S}^2 e^{2\alpha_R t}e^{2\alpha_S t}\zeta_{RS}\,. \quad (16)$$

The two above equations form the main result of the present note. Analysis of these equations is a very important problem currently being studied.

The example of perturbations with complex roots in the original linearized action (6) was carried out in [17]. Linear perturbations in such a configurations can be confined thus not destroying the system itself. This result is not obvious and it supports the claim that the SFT based models are stable. The case of complex J_P has never been studied in general and deserves deeper investigation. Analysis of perturbations with complex roots in the exactly solvable model will be addressed in a forthcoming publications.

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